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A two-parameter deformation of the Jaynes–Cummings model: path integral representation

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Abstract. A two-parameter (p,q) deformation of the Jaynes–Cummings model is obtained using a recently developed (p,q)-deformed oscillator. In the rotating wave approximation (RWA) the dynamical symmetry of the model is the quantum superalgebra $u_{p,q}(1|1)$. The partition function of the model is obtained as a path integral over generalized Perelomov coherent states corresponding to the quantum algebra $u_{p,q}(1|1)$. The complete spectrum of the model is extracted for both the case when the coupling constants are Grassmann valued and when they are c-number valued. It is noted that a relaxation of the RWA extends the dynamical symmetry to the quantum superalgebra $osp_{p,q}(2|2)$.

1. Introduction

The Jaynes–Cummings (JC) model [1] provides an idealization of the interaction of matter with electromagnetic radiation by a Hamiltonian of a two-level atom coupled to a single bosonic mode. The dynamics of the model is supported by the Rydberg maser experiments [2, 3] designed to detect the atomic coupling with a single photon. Using the Holstein–Primakoff method [4] the JC Hamiltonian may be described [5] in terms of a single fermionic mode coupled linearly with a bosonic mode. The fermion mode reflects the two-level structure of the atom. In a rotating wave approximation (RWA) a spectrum generating unitary superalgebra $u(1|1)$ describes [5] the underlying symmetry of the JC Hamiltonian. A 'dressed' version of the model relaxes the RWA and includes in the Hamiltonian the additional operators responsible for the virtual transitions and the two photon operators. The dynamical symmetry of the 'dressed' JC Hamiltonian is known [5] to be the Lie superalgebra $osp(2|2)$. Recently, using a $u(1|1)$ coherent state construction [6, 7], Kochetov [8] obtained a path integral representation of the partition function of the JC model and thereby extracted the full energy eigenspectrum for both the cases when the coupling constants are Grassmann valued and c-number valued. The eigenspectrum obtained in [8] agrees with the earlier results [5, 9].

On the other hand, the quantized algebras [10, 11] viewed as deformations of classical Lie algebras, depending, in general, on one or more deformation parameters

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are of considerable physical and mathematical interest. The representation theory of the quantum algebras with a single deformation (or quantization) parameter q led to the development of the q-deformed oscillator (q-oscillator) algebra [12–14]. Using a q-oscillator description Chaichian *et al* [15] investigated a q-deformed JC (q-JC) model with an intensity-dependent coupling. The dynamical symmetry of the q-JC Hamiltonian is described by the quantum superalgebra $\mathrm{u}_q(1|1)$. Considering the quantum algebra $\mathrm{su}_{p,q}(2)$, where p and q are two independent deformation parameters [16], the present authors obtained [17] a (p,q)-deformed oscillator ((p,q)-oscillator) realization for it. We may adopt the philosophy that quantum group inspired algebraic deformation is a way of building a model to account for small deviations, if any, from a standard one, with the deformation parameter(s) as phenomenological constant(s) to be adjusted to fit the experimental data. The existence of the two-parameter deformation implies an infinite number of one-parameter deformations with the standard case corresponding to $p = q$. Hence, it is important to investigate the consequences of this fact in model building.

Here we propose to study a (p,q)-deformed JC ((p,q)-JC) model obtained by deforming the usual Bose degree of freedom to a (p,q)-oscillator mode. In contrast to the Bose degree of freedom, the Fermi degree of freedom reflecting the two-level atomic structure is retained undeformed; when the Pauli principle is respected the deformation of an independent single-mode fermion is actually trivial [18, 19]. The dynamical symmetry of the (p,q)-JC model in the RWA may be recognized as the deformed superalgebra $\mathrm{u}_{p,q}(1|1)$, whose bosonic sector is $\mathrm{u}(1) \oplus \mathrm{u}(1)$. Following Kochetov [8], we express the partition function of the (p,q)-JC model as a path integral over generalized Perelomov coherent states corresponding to $\mathrm{u}_{p,q}(1|1)$, and subsequently extract the energy eigenspectrum for the (p,q)-JC Hamiltonian. This procedure holds for both the Grassmann and the c-valued coupling constants. In the undeformed limit ($p, q \to 1$) our results reduce to the well-known spectrum [5, 9] obtained earlier for the Grassmann and the c-valued coupling constants. In an alternate description, à la Buzano *et al* [5], we diagonalize directly the (p,q)-JC Hamiltonian for the Grassmann valued coupling constants and reobtain the corresponding energy spectrum. If the RWA is relaxed one may add to the (p,q)-JC Hamiltonian the deformed analogues of the operators responsible for the virtual and the real two-photon processes. Then, the dynamical symmetry generated by the complete set of operators is the quantum superalgebra $\mathrm{osp}_{p,q}(2|2)$.

This paper is structured as follows. In section 2, we consider the dynamical symmetry of the (p,q)-JC Hamiltonian and construct the $\mathrm{u}_{p,q}(1|1)$ coherent states. In section 3, we develop a path integral formalism over the $\mathrm{u}_{p,q}(1|1)$ coherent states and extract the energy eigenspectrum from the knowledge of the partition function. Section 4 contains our conclusions.

2. A (p,q)-JC model with the dynamical symmetry $\mathrm{u}_{p,q}(1|1)$

To set up the framework for introducing a deformed (p,q)-JC model, we first briefly review the aspects of the standard JC construction. Using the Holstein–Primakoff method [4], the JC Hamiltonian may be expressed [5] in terms of a pair of fermionic (f) and bosonic (b) modes, satisfying the super-Heisenberg algebra

$$[b, b^\dagger] = 1 \qquad \{f, f^\dagger\} = 1 \qquad f^2 = 0 \qquad f^{\dagger 2} = 0$$
$$[b, f] = 0 \qquad [b^\dagger, f] = 0\,. \tag{2.1}$$

Maintaining the RWA which entails an exclusion of the energy-non-conserving and two-photon operator terms, the JC Hamiltonian takes the form

$$H_{\mathrm{JC}} = \omega_{\mathrm{b}}\left(b^\dagger b + \tfrac{1}{2}\right) + \omega_{\mathrm{f}}\left(f^\dagger f - \tfrac{1}{2}\right) + g b f^\dagger + b^\dagger f \bar{g} \tag{2.2}$$

where the coupling constants $(g, \bar{g})$ are considered as either Grassmann or ordinary c-valued numbers. The Hamiltonian (2.2) is an element of the spectrum generating unitary superalgebra u(1|1). Defining the even and the odd generators

$$N = b^\dagger b + f^\dagger f \qquad M = b^\dagger b - f^\dagger f + 1 \qquad Q = b f^\dagger \qquad Q^\dagger = b^\dagger f\,. \tag{2.3}$$

The u(1|1) commutation relations are

$$\begin{aligned} &[N, M] = 0 && [N, Q] = 0 && [N, Q^\dagger] = 0 \\ &[M, Q] = -2Q && [M, Q^\dagger] = 2Q^\dagger && \\ &\{Q, Q^\dagger\} = N && Q^2 = 0 && Q^{\dagger 2} = 0\,. \end{aligned} \tag{2.4}$$

The relations (2.3) and (2.4) suggest that the Casimir operator N has non-negative integer eigenvalues, which label the irreducible representations (IRS) of the u(1|1). Using the notation (2.3), the Hamiltonian (2.2) reads as

$$H_{\mathrm{JC}} = \Omega N + \omega M + g Q + Q^\dagger \bar{g} \tag{2.5}$$

where

$$\Omega = \frac{\omega_{\mathrm{b}} + \omega_{\mathrm{f}}}{2} \qquad \omega = \frac{\omega_{\mathrm{b}} - \omega_{\mathrm{f}}}{2}\,. \tag{2.6}$$

To construct a (p, q)-JC model we adopt the above prescription and use a (p, q)-deformed bosonic mode $(\tilde{b}, \tilde{b}^\dagger, N_{\tilde{b}})$ obtained [17] from the study of the representation theory of the quantum algebra $\mathrm{su}_{p,q}(2)$. The corresponding deformed Heisenberg commutation relations [17] are, with p and q both real or $p\bar{q} = 1$

$$\begin{aligned} &[N_{\tilde{b}}, \tilde{b}] = -\tilde{b} && [N_{\tilde{b}}, \tilde{b}^\dagger] = \tilde{b}^\dagger \\ &\tilde{b}\tilde{b}^\dagger - q\tilde{b}^\dagger\tilde{b} = p^{-N_{\tilde{b}}} && \tilde{b}\tilde{b}^\dagger - p^{-1}\tilde{b}^\dagger\tilde{b} = q^{N_{\tilde{b}}}\,. \end{aligned} \tag{2.7}$$

The Fermi mode in (2.2) essentially reflects the two-level structure of the idealized atom and we keep it undeformed; as already mentioned above, the deformation of an independent single-mode fermionic degree of freedom is actually trivial [18] with $f^2 = f^{\dagger 2} = 0$. Maintaining the RWA structure, we consider the (p, q)-JC Hamiltonian to be given by

$$H = \Omega[\tilde{N}] + \omega[\tilde{M}] + g\tilde{Q} + \tilde{Q}^\dagger \bar{g} \tag{2.8}$$

where

$$[x] = \frac{q^x - p^{-x}}{q - p^{-1}} \tag{2.9}$$

and

$$\tilde{N} = N_{\tilde{b}} + f^\dagger f \qquad \tilde{M} = N_{\tilde{b}} - f^\dagger f + 1 \qquad \tilde{Q} = \tilde{b} f^\dagger \qquad \tilde{Q}^\dagger = \tilde{b}^\dagger f\,. \tag{2.10}$$

The dynamical symmetry of this Hamiltonian is the quantum superalgebra $u_{p,q}(1|1)$ with the commutation relations

$$\begin{array}{lll} [\tilde{N}, \tilde{M}] = 0 & [\tilde{N}, \tilde{Q}] = 0 & [\tilde{N}, \tilde{Q}^\dagger] = 0 \\ [\tilde{M}, \tilde{Q}] = -2\tilde{Q} & [\tilde{M}, \tilde{Q}^\dagger] = 2\tilde{Q}^\dagger & \\ \{\tilde{Q}, \tilde{Q}^\dagger\} = \tilde{N} & \tilde{Q}^2 = 0 & \tilde{Q}^{\dagger 2} = 0\,. \end{array} \tag{2.11}$$

It may be noted here that the deformation of a given Hamiltonian is not unique. The general guiding principle available now is the correspondence principle by which one would require the deformation to disappear in the appropriate limit. Thus, in (2.8) one may take the bosonic part to be an arbitrary function of $\tilde{N}$ and $\tilde{M}$ with the correct limiting behaviour. The procedure outlined below to obtain the specturm of H in (2.8) is quite general and can also be adopted when one has arbitrary functions of $\tilde{N}$ and $\tilde{M}$ in the bosonic part.

For the algebra (2.11) we choose the non-negative integer Casimir operator $\tilde{N}$ and $f^\dagger f\ (= (\tilde{N} - \tilde{M} + 1)/2\,)$ as the complete set of commuting operators. Selecting an integer $n > 0$, we choose the basis states $\{\,|n, I\rangle \mid I = (0,1)\,\}$ of the IR of $u_{p,q}(1|1)$ as

$$\tilde{N}|n\,, I\rangle = n|n\,, I\rangle \qquad f^\dagger f|n\,, I\rangle = I|n\,, I\rangle\,. \tag{2.12}$$

For $n = 0$, the IR is one-dimensional

$$\tilde{N}|0,0\rangle = 0 \qquad f^\dagger f|0,0\rangle = 0\,. \tag{2.13}$$

The other generators act on the basis states as

$$\begin{gathered} \tilde{M}|n, I\rangle = (n - 2I + 1)|n\,, I\rangle \\ \tilde{Q}|n, I\rangle = (1 - \delta_{n,0})\delta_{I,0}\sqrt{[n]}|n, I+1\rangle \qquad \tilde{Q}^\dagger|n, I\rangle = \delta_{I,1}\sqrt{[n]}|n, I-1\rangle\,. \end{gathered} \tag{2.14}$$

For the choice of the coupling constants $(g, \bar{g})$ as Grassmann numbers, we diagonalize the Hamiltonian (2.8), à la Buzano *et al* [5], and obtain the energy eigenspectrum. To this end, we use a similarity transformation

$$H_{\mathrm{diag}} = \exp(\tilde{z})\, H \exp(-\tilde{z}) \tag{2.15}$$

where

$$\tilde{z} = -(\omega([n+1] - [n-1]))^{-1}(gQ - Q^\dagger \bar{g})\,. \tag{2.16}$$

The corresponding energy eigenvalues are

$$E_{\rm G}^{0} = \omega \tag{2.17}$$

and

$$E_{{\rm G},\pm}^{n>0} = \Omega[n] + \omega[n \pm 1] + \bar{g}g\frac{[n]}{\omega([n+1]-[n-1])}\,. \tag{2.18}$$

The spectrum given by (2.17) and (2.18) agrees with the known result [5] in the undeformed $(p,q \to 1)$ limit.

We will devote the remaining part of the present section to the construction of the Perelomov-type coherent states for the quantum algebra $\mathrm{u}_{p,q}(1|1)$. In the sector $n > 0$, let us define the $\mathrm{u}_{p,q}(1|1)$ coherent state as

$$|n;\theta\rangle = \exp(-\theta\tilde{Q}^{\dagger})|n,1\rangle = |n,1\rangle - \sqrt{[n]}\theta|n,0\rangle\,. \tag{2.19}$$

It is easily seen that

$$Q|n;\theta\rangle = [n]\theta|n;\theta\rangle\,. \tag{2.20}$$

The overlap of two states is

$$\langle n;\zeta|n;\theta\rangle = \exp([n]\bar{\zeta}\theta)\,. \tag{2.21}$$

The integration rules for the Grassmann variables are as follows

$$\int \mathrm{d}\theta = 0 \qquad \int \mathrm{d}\bar{\theta} = 0 \qquad \int \mathrm{d}\theta\,\theta = 1 \qquad \int \mathrm{d}\bar{\theta}\,\bar{\theta} = 1\,. \tag{2.22}$$

For $n > 0$, the completeness relation

$$\sum_{I=0,1} |n,I\rangle\langle n,I| = \mathbf{1}_n \tag{2.23}$$

reads in the coherent state description

$$\int \frac{\mathrm{d}\bar{\theta}\,\mathrm{d}\theta}{[n]} \exp(-[n]\bar{\theta}\theta)|n;\theta\rangle\langle n;\theta| = \mathbf{1}_n\,. \tag{2.24}$$

The trace of an operator in the two-dimensional space $\{\,|n,I\rangle \mid n > 0, I = 0,1\,\}$ has the coherent states representation

$$\mathrm{Tr}_n\{\hat{O}\} = \sum_{I=0,1} \langle n,I|\hat{O}|n,I\rangle = \int \frac{\mathrm{d}\theta\,\mathrm{d}\bar{\theta}}{[n]} \exp([n]\bar{\theta}\theta)\langle n;\theta|\hat{O}|n;\theta\rangle\,. \tag{2.25}$$

In the first equality in (2.25), the suffix 'n' in the LHS denotes a summation over IR with a fixed value of $n(>0)$. For use later, we obtain here the following matrix element of the Hamiltonian (2.8)

$$\begin{aligned} H(\bar{\zeta},\theta) &= \frac{\langle n;\zeta|H|n;\theta\rangle}{\langle n;\zeta|n;\theta\rangle} \\ &= \Omega[n] + \omega[n-1] + [n](g\theta + \bar{\zeta}\bar{g}) + \omega[n]([n+1]-[n-1])\bar{\zeta}\theta\,. \end{aligned} \tag{2.26}$$

3. A functional integral over the $u_{p,q}(1|1)$ coherent states

In this section, we develop, à la Kochetov [8], a path integral description of the (p,q)-JC partition function. The partition function is expressed as a sum over the partial traces

$$Z = \mathrm{Tr}\{e^{-\beta H}\} = Z_o + \sum_{n=1}^{\infty} Z_n \tag{3.1}$$

where

$$Z_o = \langle 0,0|\exp(-\beta H)|0,0\rangle = e^{-\beta\omega} \tag{3.2}$$

$$Z_n = \mathrm{Tr}_n\{e^{-\beta H}\} = \int \frac{d\theta\, d\bar{\theta}}{[n]} \exp([n]\bar{\theta}\theta)\langle n;\theta|\exp(-\beta H)|n;\theta\rangle\,. \tag{3.3}$$

Using the standard construction for the path integral for an imaginary time interval, we write

$$Z_n = \lim_{\mathcal{N}\to\infty} \int \prod_{j=0}^{\mathcal{N}} \left(\frac{d\bar{\zeta}_j\, d\zeta_j}{[n]} \exp(-[n]\bar{\zeta}_j\zeta_j)\right) E(\bar{\zeta}_o, \zeta_{\mathcal{N}}) \times \prod_{k=1}^{\mathcal{N}} \langle n;\zeta_k|\exp(-\epsilon H)|n;\zeta_{k-1}\rangle \tag{3.4}$$

where

$$\epsilon = \beta/\mathcal{N} \tag{3.5}$$

$$E(\bar{\zeta}_o,\zeta_{\mathcal{N}}) = \int \frac{d\theta\, d\bar{\theta}}{[n]} \exp([n]\bar{\theta}\theta)\langle n;\theta|n;\zeta_{\mathcal{N}}\rangle\langle n;\zeta_o|n;\theta\rangle\,. \tag{3.6}$$

A scale transformation

$$\zeta_k \to \frac{\zeta_k}{\sqrt{[n]}} \qquad \bar{\zeta}_k \to \frac{\bar{\zeta}_k}{\sqrt{[n]}} \tag{3.7}$$

and suitable integrations of the Grassmann variables yield

$$Z_n = \lim_{\mathcal{N}\to\infty} \int_{\zeta_o+\zeta_{\mathcal{N}}=0} \prod_{j=1}^{\mathcal{N}} d\bar{\zeta}_j\, d\zeta_j\ \exp\left\{-\epsilon \sum_{k=1}^{\mathcal{N}} \left(\bar{\zeta}_k \left(\frac{\zeta_k - \zeta_{k-1}}{\epsilon}\right) + \Omega[n] + \omega[n-1] + \sqrt{[n]}(g\zeta_{k-1} + \bar{\zeta}_k\bar{g}) + \omega([n+1]-[n-1])\bar{\zeta}_k\zeta_{k-1}\right)\right\}. \tag{3.8}$$

In the continuum limit

$$Z_n = \int_{\zeta(0)+\zeta(\beta)=0} D\bar{\zeta}(t)D\zeta(t)\exp\left\{-\int_0^{\beta} dt\,(\bar{\zeta}(t)\dot{\zeta}(t) + \Omega[n] + \omega[n-1] + \omega([n+1]-[n-1])\bar{\zeta}(t)\zeta(t) + \sqrt{[n]}(\bar{\zeta}(t)\bar{g} + g\zeta(t))\right\}. \tag{3.9}$$

For the case of Grassmann valued coupling constants $(g, \bar{g})$ the path integral (3.9) may be readily diagonalized by the following transformations

$$\begin{aligned} \zeta(t) &\to \zeta(t) - \frac{\sqrt{[n]}}{\omega([n+1]-[n-1])}\bar{g} \\ \bar{\zeta}(t) &\to \bar{\zeta}(t) - \frac{\sqrt{[n]}}{\omega([n+1]-[n-1])}g\,. \end{aligned} \tag{3.10}$$

The transformation (3.10), reduces (3.9) to the form

$$Z_n = \exp\left\{-\beta\left(\Omega[n] + \bar{g}g\frac{[n]}{\omega([n+1]-[n-1])}\right)\right\}\mathrm{Tr}_n\{\exp(-\beta\omega[M])\} \tag{3.11}$$

where

$$\begin{aligned} \mathrm{Tr}_n\{\exp(-\beta\omega[M])\} = \int_{\zeta(0)+\zeta(\beta)=0} D\bar{\zeta}(t)\,D\zeta(t)\exp\left(-\beta\omega\frac{[n+1]+[n-1]}{2}\right) \\ \times\exp\left(-\int_0^\beta \mathrm{d}t\,\bar{\zeta}(t)(\dot{\zeta}(t) + \omega([n+1]-[n-1])\zeta(t)) \right. \\ \left. + \omega\frac{[n+1]-[n-1]}{2}\beta\right)\,. \end{aligned} \tag{3.12}$$

A direct evaluation of the partial trace in (3.11) immediately leads to the energy eigenspectrum obtained in (2.18).

For the case of c-number coupling constants $(g, \bar{g})$ we rewrite (3.9) as

$$\begin{aligned} Z_n = \exp\left\{-\beta\left(\Omega[n] + \omega\frac{[n+1]+[n-1]}{2}\right)\right\}\int_{\zeta(0)+\zeta(\beta)=0} D\bar{\zeta}(t)\,D\zeta(t) \\ \times\exp\left\{-\int_0^\beta \mathrm{d}t\,\bar{\zeta}(t)(\dot{\zeta}(t) + \omega([n+1]-[n-1])\zeta(t)) \right. \\ \left. - \sqrt{[n]}\int_0^\beta \mathrm{d}t\,(\bar{\zeta}(t)\bar{g} + g\zeta(t)) + \omega\frac{[n+1]-[n-1]}{2}\beta\right\}\,. \end{aligned} \tag{3.13}$$

Following the route adopted in [8], we compare (3.13) with the partition function for a single spin-1/2 particle in a constant magnetic field, described by the Hamiltonian

$$H_0 = \Omega_0\sigma_3 + g_0\sigma_+ + \sigma_-\bar{g}_0 \tag{3.14}$$

where the coupling constants $(g_0, \bar{g}_0)$ are c-numbers. The energy eigenvalues of the Hamiltonian (3.14) are

$$E_\pm = \pm\sqrt{\Omega_0^2 + |g_0|^2} \tag{3.15}$$

and the corresponding partition function in the path integral representation [20] reads

$$\begin{aligned} Z_0 = \int_{\zeta(0)+\zeta(\beta)=0} D\bar{\zeta}(t)D\zeta(t)\exp\left\{-\int_0^\beta \mathrm{d}t\,\bar{\zeta}(t)\,(\dot{\zeta}(t) + 2\Omega\zeta(t)) \right. \\ \left. - \int_0^\beta \mathrm{d}t\,\left(g_0\zeta(t) + \bar{\zeta}(t)\bar{g}_0\right) + \Omega_0\beta\right\}\,. \end{aligned} \tag{3.16}$$

A comparison between (3.11) and (3.16) immediately yields the spectrum corresponding to Z_n as

$$E_{\pm}^{n>0} = \Omega[n] + \omega\frac{[n+1]+[n-1]}{2} \pm \left[\omega^2\left(\frac{[n+1]-[n-1]}{2}\right)^2 + [n]|g|^2\right]^{1/2} . \tag{3.17}$$

The above list and the trivial result

$$E^0 = \omega \tag{3.18}$$

obtained from (3.2) enumerate all the energy eigenvalues of the Hamiltonian (2.18) corresponding to c-valued coupling constants. Expanding the square root in (3.17) and retaining terms upto $O(|g|^2)$ we obtain

$$E_{\pm}^{n>0} \simeq \Omega[n] + \omega[n\pm 1] \pm |g|^2\frac{[n]}{\omega([n+1]-[n-1])} . \tag{3.19}$$

We notice that the spectrum in (3.19) is similar to (2.18) apart from the fact that in the latter case Grassmann properties of the coupling constants are to be taken into account.

It should be noted that even if we choose the bosonic part of the (p,q)-JC Hamiltonian to be an arbitrary function of $(\tilde{N},\tilde{M})$ then, as already mentioned, precisely the same procedure as outlined above for the case of (2.8) will hold. Here we enlist the results corresponding to an alternate Hamiltonian

$$H' = \Omega\tilde{N} + \omega\tilde{M} + g\tilde{Q} + \tilde{Q}^{\dagger}\bar{g} . \tag{3.20}$$

For the Grassmann valued coupling constants, the energy eigenvalues are

$$E'^{n>0}_{G,\pm} = \Omega n + \omega(n\pm 1) + \frac{[n]}{2\omega}\bar{g}g \tag{3.21}$$

and the spectrum for the case of c-number coupling constants reads

$$E'^{n>0}_{\pm} = (\Omega+\omega)n \pm \sqrt{w^2 + [n]|g|^2} . \tag{3.22}$$

4. Conclusion

Deforming the standard JC model we have constructed a two-parameter (p,q)-JC model and, by employing a path integral realization of the partition function, derived the full energy spectrum of the model. The existence of a dynamical symmetry algebra in the parent model allows the deformation of the symmetry algebra. We feel that it is worth investigating the consequences of deformations of solvable models with known dynamical symmetries so that one can know what signatures of deformed symmetries to look for in order to identify them if they occur in nature.

To conclude let us note the following: relaxing the RWA, we may write a 'dressed' (p,q)-JC model where the following additional operators, deformed analogues of the operators responsible for the virtual and the real two photon transitions, may contribute to the Hamiltonian

$$\tilde{K} = \tilde{b} f \qquad \tilde{K}^{\dagger} = \tilde{b}^{\dagger} f^{\dagger} \qquad \tilde{J} = \tilde{b}^{2} \qquad \tilde{J}^{\dagger} = \tilde{b}^{\dagger 2} . \tag{4.1}$$

The operators in (2.10) and (4.1) generate a new dynamical quantum superalgebra, $\mathrm{osp}_{p,q}(2|2)$, which embeds $\mathrm{u}_{p,q}(1|1)$. The additional commutation relations for $\mathrm{osp}_{p,q}(2|2)$ are

$$\begin{aligned}
&[\tilde{N}, \tilde{J}] = -2\tilde{J} \qquad [\tilde{N}, \tilde{K}] = -2\tilde{K} \\
&[\tilde{M}, \tilde{J}] = -2\tilde{J} \qquad [\tilde{M}, \tilde{K}] = 0 \qquad \tilde{K}^2 = 0 \\
&[\tilde{J}, \tilde{K}] = 0 \qquad [\tilde{J}, \tilde{Q}] = 0 \\
&[\tilde{Q}, \tilde{K}] = \tilde{J} \qquad [\tilde{Q}, \tilde{K}^{\dagger}] = 0 \qquad [\tilde{K}, \tilde{K}^{\dagger}] = [\tilde{M}] \\
&[\tilde{J}, \tilde{J}^{\dagger}] = \left[\frac{\tilde{N}+\tilde{M}+3}{2}\right]\left[\frac{\tilde{N}+\tilde{M}+1}{2}\right] - \left[\frac{\tilde{N}+\tilde{M}-3}{2}\right]\left[\frac{\tilde{N}+\tilde{M}-1}{2}\right] \\
&[\tilde{J}, \tilde{K}^{\dagger}] = \tilde{Q}\left[\frac{\tilde{N}+\tilde{M}+1}{2}\right] - \left[\frac{\tilde{N}+\tilde{M}-1}{2}\right]\tilde{Q} \\
&[\tilde{J}, \tilde{Q}^{\dagger}] = \tilde{K}\left[\frac{\tilde{N}+\tilde{M}+1}{2}\right] - \left[\frac{\tilde{N}+\tilde{M}-1}{2}\right]\tilde{K} .
\end{aligned} \tag{4.2}$$

From (2.11) and (4.2) we notice that the operators $(\tilde{M}, \tilde{N}, \tilde{K}, \tilde{K}^{\dagger})$ form a quantum subalgebra $\mathrm{u}_{p,q}(1|1) \subset \mathrm{osp}_{p,q}(2|2)$, different from (2.11) with $\tilde{M}$ as the new Casimir operator.

Further, one may also consider the basic boson and fermion oscillators of the JC model to be deformed into a pair of supersymmetric oscillators covariant under a quantum supergroup [21]. Then, the resulting model is different; in particular the deformation of the fermionic mode now has a non-trivial consequence.

We shall return to this topic in future work.

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